Portfolio Selection and Optimization

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Abstract
The problem of optimal allocation of resources and portfolio selection is considered in this work. To achieve this objective which will offer investors the opportunity to invest their limited available resource wisely for optimal return, we derived optimal weights for two, three and four stocks using expected returns and minimum variance approach. Data from Diamond bank, Eco bank, First bank, UBA and Zenith bank were studied and results show that Eco bank has the highest return 2.8% (log-return) and -0.9% (return at time t) while First bank has the lowest returns. The variance of the return computed show that First Bank has the lowest variance while Eco bank has the highest variance. The proportion of fund to be invested in each of the securities based on their combination was calculated. With expected returns and variance of the portfolio, First bank and Zenith bank have most attraction. We observed that as more securities are combined the lesser the variance compared with their individual securities.

Key Words: *Portfolio Selection, Resources, Securities, Diversifications and Optimization.*

1.0 Introduction

Active trading strategies are basically targeted at beating the underlying benchmark. With the limited available resources (capital) a man may wish to invest in a number of securities. Out of N available securities, which of the stocks will he invest his limited resource in order to obtain optimal return? This is a problem of portfolio selection and optimization.

Modern portfolio theory is a mathematical formulation of the concept of diversification in investing, with the aim of selecting a collection of investment assets that have collectively lower risk than any individual asset. This theory can also be seen as Harry Markowitz theory (Markowitz, 1952) of portfolio choice in an uncertain future. In his work he quantified the difference between the risk of portfolio assets taken individually and the overall risk of the portfolio. The optimal portfolio from a rational investor's point of view is defined as those that have the lowest risk for a given return. Investing in the stock market always bears some risk, large or small, depending on the variance of the returns of the stock. A stock that has large variance may make you rich but may also make you poor. Therefore, risk is synonymous with variance and if investors are risk averse they will like to minimize risk. Suppose an investor has two stocks and wants to make an investment, there are many possibilities. One of them is to invest all his available funds in the first stock and nothing in the second or vice versa. Another possibility is to invest 50% of his funds in the first stock and another 50% in the second stock (equal allocation), etc. Which investment should we choose?

A risk averse investor would like to minimize his/her risk, therefore investing in a stock with minimum variance of the portfolio. If A and B are two stocks and variance of return of stock

A is greater than variance of return of stock B (i.e. Var (R_A)>Var (R_B)) it means that stock A is riskier than that of B. A risk averse investor would want to invest more in stock B than in A in order to minimize the risk of the portfolio.

In this study we explore the building of efficient portfolio through optimization using examples for two, three and four stocks. The relevance of this study lies on the fact that many people invest their limited available resource into a number of securities without considering the risk associated with the expected return. They end up investing in securities that have high covariance with each other which does not provide much (efficient) diversification. In most cases their method of allocating funds to each of these securities are not guided by any factor. Hence need for portfolio management.

The ultimate objective of this study is to build an efficient portfolio through optimization using examples of four stocks and how covariance and correlation can help investor diversify their risk. We shell therefore aimed at deriving the general formula for calculating the weights of the securities in a portfolio with two, three and four stocks. It also aimed at educating investors on the factors to be considered for efficient portfolio. Through this study, investors can wisely select n stocks from N listed companies so as to minimize risk and maximize output. The determination of optimal weights that will lead to optimum allocation of capital into the n selected companies, which in turn maximize return and minimize risk, is also possible.

This study shall be limited to four (4) companies quoted in the Nigeria stock exchange and their monthly closing stock prices for the last four (4) years.

1.1 Definition of Terms

1.1.1 Log Returns

Let $P_{i1}, P_{i2},\ldots, P_{in}$ be the prices of the i-th stock (i = A,B,C,D) and R_A, R_B, R_C, R_D be the returns of the four stocks. Benninga (1997) defined log return as

$$R_{it} = \ln\left(\frac{P_{it}}{P_{it-1}}\right) \quad \text{for } i = 1,2,3,4 \qquad (1.1)$$

Where $P_{i,t}$is the t-th stock price of the i-th portfolio
$P_{i,t-1}$ is the (t-1)th stock price of the i-th portfolio
R_{it} is the t-th log return of the i-th portfolio.

1.1.2 Returns at Time (t)

Let $P_{i1}, P_{i2},\ldots,P_{in}$ be the prices of i-th stock (i = A, B, C, D) and R_A, R_B, R_C, R_D be the returns of the four stocks, then $R_{i,t} = \frac{P_t - P_{t-1}}{P_{t-1}}$ for i = 1, 2,3, 4 (1.2)

Where $P_{t,,}$ P_{t-1} are the closing prices of stocks at time t and t-1 respectively and R_{it} is the return of i-th stock.

1.1.3 Mean of the Returns

We make a heroic assumption. We assume that the data collected represent the distribution of the return for the coming month. That is the past gives us some information about the way returns will behave in the future. This assumption allows us to assume that the average of the historic data represents the expected return from each stock. Also we can learn what the variance of the future returns is, using historic data.
The mean of the i-th return (i = R_A, R_B, R_C, R_D) is given as;

$\overline{R}_{it} = \frac{\sum_{t-1}^{n} R_{it}}{n}$ for i = 1,2,3,4. (1.3)

where n = the number of observations and t = the time

1.1.4 Variance of the Returns

Let $R_{i1}, R_{i2},\ldots,R_{in}$ be the returns of a company at time t, then variance of returns

$\text{Var}(R_i) = \sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(R_{it} - \overline{R}_t)^2$ (1.4)

where n = number of observation

R_{it} = the returns of i-th stock at time t while $\overline{R}_t$ = the mean of return

For log return, let $\ln R_{i1}, \ln R_{i2},\ldots,\ln R_{in}$ be the log returns of a company at time t, then variance of the log returns is

$\text{Var}(R_{it}) = \frac{1}{n-1}\sum_{t=1}^{n}(InR_{it} - \overline{R}_t)^2$ (1.5)

where n = the number of observations

$\overline{R}_t$ = the mean of the log returns

The standard deviations are the square roots of the variances, that is

$\sigma_i = \sqrt{V(R_i)}$ (1.6)

1.1.5 Covariance of Returns

It is a measure of the degree to which returns on two risky assets move in tandem.

A positive covariance means that assets returns move together. A negative covariance means returns move inversely.

Processing financial assets that provide returns and have a high covariance with each other will not provide very much diversification. If an investor wants a portfolio whose assets have diversified earnings, he or she should pick financial assets that have low covariance to each other.

The covariance between the returns of stocks R_A, R_B, R_C, R_D is given as

$\text{Cov}(R_A, R_B, R_C, R_D) = \sigma_{ABCD} = \frac{1}{t-1}\sum_{t=1}^{n}(R_{At} - \overline{R}_A)(R_{Bt} - \overline{R}_B)(R_{Ct} - \overline{R}_C)(R_{Dt} - \overline{R}_D)$ (1.7)

For log return, covariance between the returns of stocks, A, B, C, D is

$\text{Cov}(\ln R_{At}, \ln R_{Bt}, \ln R_{Ct}, \ln R_{Dt}) = \sigma_{ABCD} = \frac{1}{t-1}\sum_{t=1}^{n}(InR_{At} - \overline{R}_A)(InR_{Bt} - \overline{R}_B)(InR_{Ct} - \overline{R}_C)$

$(InR_{Dt} - \overline{R}_D)$ (1.8)

1.1.6 Correlation Coefficient

The correlation coefficient measures the degree of linear relationship between the returns of stock A, B, C & D

$\rho_{ABCD} = \text{Cor}\frac{R_A R_B R_C R_D}{\sigma_A \sigma_B \sigma_C \sigma_D}$ (1.9)

2.0 Methodology

We are interested in determining the proportion of fund to invest in stocks in order to maximize returns and minimize risk using Log and return at time (t).

2.1 Derivation of Portfolio Management for Two Stocks, Three Stocks and Four Stocks

2.1.1 Portfolio Management with Two Stocks

Christou (2008) states that if R_A and R_B are the returns of two stocks A and B, W_A and W_B the proportion of available funds to be invested in each of the stocks then the resulting portfolio will be $w_AR_A + w_BR_B$

A risk averse investor will like to minimize his/her risk, therefore minimizing the variance of the portfolio i.e.

$Var(w_AR_A + w_BR_B) = w^2_AVar(R_A) + w^2_BVar(R_B) + 2w_Aw_BCov(R_AR_B)$ (2.1)

Which is subject to budget constraint?

$w_A + w_B = 1$ which implies that

$w_B = 1 - w_A$

Let $Q = w^2_AVar(R_A) + (1-w_A)^2Var(R_B) + 2w_A(1-w_A)\ Cov\ R_AR_B$ (2.2)

Let also $Var(R_A) = V_A$

$Var(R_B) = V_B$

$Cov(R_A,R_B) = V_{AB}$

It implies that $Q = w^2_AV_A + (1-w_A)^2V_B + 2w_A(1-w_A)V_{AB}$ (2.3)

Minimizing the variance of the portfolio, we differentiate Q w.r.t w_A, w_B respectively and equate to zero.

$\frac{\partial Q}{\partial w_A} = 2w_AV_A - 2(1 - w_A)\ V_B + [2(1 - w_A) - 2w_A]V_{AB} = 0$

Simplifying and making w_A the subject, we have

$w_A = \frac{V_B - V_{AB}}{V_A + V_B - 2V_{AB}}$ (2.4)

$\hat{w}_B = \frac{V_A - V_{AB}}{V_A + V_B - 2V_{AB}}$ (2.5)

2.1.2 Portfolio Management with Three Stocks

Let w_A, $w_B$and W_C be the proportion of the available funds invested in the three stocks A, B and C respectively and R_A,R_B and R_C be the returns of stock A, B and C. The resulting portfolio will be;

$w_AR_A + w_BR_B + w_CR_C$

where $w_A + w_B + w_C = 1$ (2.6)

The variance of the portfolio is

$Var(w_AR_A + w_BR_B + w_CR_C) = w^2_A\ V_A + w^2_BV_B + w^2_CV_C + 2\ w_Aw_BCovAB + 2w_Aw_CCovAC + 2w_Bw_CCovBC$

$= w^2_A\ V_A + w^2_BV_B + w^2_CV_C + 2\ w_Aw_BV_{AB} + 2w_Aw_CV_{AC} + 2w_Bw_CV_{BC}$

But $w_C = 1 - w_A - w_B$

$w^2_A\ V_A + w^2_BV_B + (1 - w_A - w_B)^2V_C + 2\ w_Aw_BV_{AB} + 2w_A(1 - w_A - w_B)V_{AC} + 2w_B(1 - w_A - w_B)V_{BC}$

Let $Q = w^2_A\ V_A + w^2_BV_B + (1 - w_A - w_B)^2V_C + 2w_Aw_BV_{AB} + 2w_A(1 - w_A - w_B)V_{AC} + 2w_B(1 - w_A - w_B)V_{BC}$ (2.7)

Minimizing the variance of the portfolio, we differentiate Q w.r.t w_A, w_B, and w_C respectively and equate to 0. Putting the in matrix for, we have

$$\begin{bmatrix} (V_A + V_C - 2V_{AC}) & (V_C + V_{AB} - V_{AC} - V_{BC}) \\ (V_C + V_{AB} - V_{AC} - V_{BC}) & (V_B + V_C - 2V_{BC}) \end{bmatrix} \begin{bmatrix} w_A \\ w_B \end{bmatrix} = \begin{bmatrix} (V_C - V_{AC}) \\ (V_C - V_{BC}) \end{bmatrix} \quad (2.8)$$

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} w_A \\ w_B \end{bmatrix}=\begin{bmatrix} m \\ n \end{bmatrix} \quad (2.9)$$

Where $a = (V_A + V_C - 2V_{AC})$

$b = (V_C + V_{AB} - V_{AC} - V_{BC})$

$c = (V_B + V_C - 2V_{BC})$

$m = (V_C - V_{AC})$

$n = (V_C - V_{BC})$

$$\begin{bmatrix} \hat{w}_A \\ \hat{w}_B \end{bmatrix} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}^{-1}\begin{bmatrix} m \\ n \end{bmatrix}$$

$Det(D) = ac - b^2$

$$\begin{bmatrix} \hat{w}_A \\ \hat{w}_B \end{bmatrix} = \frac{1}{D}\begin{bmatrix} c & -b \\ -b & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}$$

$\hat{w}_A = \frac{Cm-bn}{D}$, $\hat{w}_B = \frac{an-bm}{D}$ and $\hat{w}_c = 1 - \hat{w}_A - \hat{w}_B$ (2.10)

$\hat{w}_C = 1 - \frac{cm-bn}{ac-b^2} - \frac{an-bm}{ac-b^2}$

$= \frac{ac-cm+bn-an+bm-b^2}{ac-b^2}$

2.1.3 Portfolio Management with Four Stocks

Let w_A, w_B, w_C and w_D be the proportion of the available funds invested in the four stocks A, B, C and D respectively and R_A, R_B, R_C and R_D be the returns of stock A, B, C and D, then the resulting portfolio will be;

$w_AR_A + w_BR_B + w_CR_C + w_DR_D$

The variance of the portfolio is

$Var(w_AR_A + w_BR_B + w_CR_C + w_DR_D) = w^2_AV_A + w^2_BV_B + w^2_CV_C + w^2_DV_D + 2w_Aw_BV_{AB} + 2w_Aw_CV_{AC} + 2w_Aw_DV_{AD} + 2w_Bw_CV_{BC} + 2w_Bw_DV_{BD} + 2w_Cw_DV_{CD})$

Let $Q = w^2_AV_A + w^2_BV_B + w^2_CV_C + w^2_DV_D + 2w_Aw_BV_{AB} + 2w_Aw_CV_{AC} + 2w_Aw_DV_{AD} + 2w_Bw_CV_{BC} + 2w_Bw_DV_{BD} + 2w_Cw_DV_{CD})$

This is subject to the constraint

$W_A + w_B + w_C + w_D = 1$

It implies that $W_D = 1 - w_A - w_B - w_C$.

$Q = w^2_AV_A + w^2_BV_B + w^2_CV_C + (1 - w_A - w_B - w_C)^2V_D + 2w_Aw_BV_{AB} + 2w_Aw_CV_{AC} + 2w_A(1 - w_A - w_B - w_C)V_{AD} + 2w_Bw_CV_{BC} + 2w_B(1 - w_A - w_B - w_C)V_{BD} + 2w_C(1 - w_A - w_B - w_C)V_{CD}$ (2.11)

Minimizing the variance of the portfolio, we differentiate Q with respect to w_A, w_B, and w_C and equate to zero. By simplifying and putting the result in matrix form, we have

$$\begin{bmatrix} (V_A + V_D - 2V_{AD}) & (V_D + V_{AB} - V_{AD} - V_{BD}) & (V_D + V_{AC} - V_{AD} - V_{CD}) \\ (V_D + V_{AB} - V_{AD} - V_{BD}) & (V_B + V_D - 2V_{BD}) & (V_D + V_{BC} - V_{BD} - V_{CD}) \\ (V_D + V_{AC} - V_{AD} - V_{CD}) & (V_D + V_{BC} - V_{BD} - V_{CD}) & (V_C + V_D - 2V_{CD}) \end{bmatrix}\begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix}$$

$$= \begin{bmatrix} (V_D - V_{AD}) \\ (V_D - V_{BD}) \\ (V_D - V_{CD}) \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix} = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

Where $a = (V_A + V_D - 2V_{AD})$
$b = (V_D + V_{AB} - V_{AD} - V_{BD})$
$c = (V_D + V_{AC} - V_{AD} - V_{CD})$
$d = (V_B + V_D - 2V_{BD})$
$e = (V_D + V_{BC} - V_{BD} - V_{CD})$
$f = (V_C + V_D - 2V_{CD})$
$l = (V_D - V_{AD})$
$m = (V_D - V_{BD})$
$n = (V_D - V_{CD})$

Calculating the determinant, we have
$Det(D) = adf-ae^2-fb^2+2ceb-de^2$

$$\begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix} = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}^{-1} \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

Solving the above matrix, we have

$$\begin{bmatrix} \hat{w}_A \\ \hat{w}_B \\ \hat{w}_C \end{bmatrix} = \begin{bmatrix} (df - e^2) & (ce - bf) & (be - cd) \\ (ce - bf) & (af - c^2) & (cb - ae) \\ (be - cd) & (cb - ae) & (ad - b^2) \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

$$\hat{w}_A = \frac{ldf - le^2 + mce - mbf + ben - ncd}{D} \quad (2.15)$$

$$\hat{w}_B = \frac{lec - lbf + fam - mc^2 + ben - ean}{D} \quad (2.16)$$

$$\hat{w}_C = \frac{lbe - lcd + mcb - mae + nad - nb^2}{D} \quad (2.17)$$

$$\hat{w}_D = 1 - \hat{w}_A - \hat{w}_B - \hat{w}_C \quad (2.18)$$

3.0 Estimation of the Expected Return of Portfolio

Let w_A, w_B, w_C, w_D be the proportion of total portfolio funds invested in stock A,B,C and D then the

Expected Return $(E_p) = E(w_A) + E(w_B) + E(w_C) + E(w_D)$
$= E(w_A R_A + w_B R_B + w_C R_C + w_D R_D)$ (3.1)

Where; E_p = the expected return on the entire portfolio
R_A = the return of stock A
R_B = the return of stock B
R_C = the return stock C
R_D = the return of stock D

3.1 Estimation of the Variance of Portfolio

The variance of the portfolio (VP) $= Var[(w_A(R_A) + w_B(R_B) + w_C(R_C) + w_D(R_D)]$
$= w_A^2 V_A + w_B^2 V_B + w_C^2 V_C + w_D^2 V_D + 2w_A w_B V_{AB} + 2w_A w_C V_{AC} + 2w_A w_D V_{AD}$
where
w_A = the proportion of the fund to be invested in stock A

w_B = the proportion of the fund to be invested in stock B
w_C = the proportion of the fund to be invested in stock C
w_D = the proportion of the fund to be invested in stock D
R_A, R_B, R_C& R_D are as defined above.

4.1 Data Analysis

The data for this research work were collected from the website of https://docs.google.com/file/d/0B2ZON4IwAInUYmJhZDAwYzMtNzE2Yi00MjU4LTliMDAtMzcxN2FlOTIxNTky/edit?hl=en_US&pli=1 .They were closing stock prices of five banks namely; First bank, Diamond bank, Eco bank, United bank for Africa (UBA) and Zenith bank. It is a four-year daily data, from January 2008 to December 2011 making a total of 1152 trading days. These were then converted to monthly data. Since according to Nicolas Christou (Journal of statistics Education volume 16n3) monthly returns are usually use in portfolio management. Therefore for the purpose of this study a forty-eight monthly data for each of the five stocks was used. The data displayed in appendix 1.

4.2 The Prices of Stocks

Appendix 1 shows the monthly stock prices of the five banks while Appendix II, fig 1 (1a-1e) shows the prices movement graph of the above companies. The fluctuations (movement) of the prices make the market uncertain. From the graphs, we observed that First bank and Zenith bank have the best trend, their graphs moves slowly over time.
Eco bank has the most uncertain prices. An investor would invest in any one of them based on the investor's preference.

For a risk averse investor (an investor that likes a certain return to uncertain ones) will like to invest in First bank then Zenith bank since they have the lowest volatility (variance) respectively. On the other hand, a risk loving investor will prefer to invest in Eco bank because it has the highest volatility (variance).

4.3 The Return

The log returns and return at time t were calculated using equations (1.1) and (1.2) respectively and the data displayed in Appendices II. The plots for log returns are shown in figure 1 of Appendix II. From the plots the month of September is highly a volatile month in that prices rise or fall arbitrarily.

4.4 Mean Return

The mean returns were calculated using equation (1.3) and it is shown below

Table 3.1 Mean of the return

	Log return	**Return at time (t)**
First bank	- 0.021104829	- 0.01581
Diamond bank	- 0.004644811	0.0079226
Ecobank	- 0.00936	0.027582
UBA	- 0.025150094	- 0.01327
Zenith	- 0.013306543	- 0.00426

Table 3.1 shows that the average monthly prices of First bank have – 0.01581 which is – 1.581 percent decrease and its monthly log rate is – 0.021104529, in case of Diamond bank, the monthly price percentage rate increase is 0.79226 while its log rate is – 0.004644811. Those of Ecobank, UBA and Zenith bank are as shown in the table above.

4.5 Variance and Standard Deviation of Return

The variance and standard deviation of the returns were calculated using equation (1.4) and (1.5).

Table 3.2a Variance of the Return

	Log return	**Return at time (t)**
First bank (FB)	0.010441461	0.009889
Diamond bank (DB)	0.023107733	0.0305648
Ecobank (EB)	0.053866	0.157747
UBA (UB)	0.023665057	0.023648
Zenith bank	0.017722265	0.018918

Table 3.2b Standard Deviation of the Return

	Log σ	**σ of Return at time (t)**
First bank	0.102183466	0.099445
Diamond bank	0.15201228	0.1748279
Ecobank	0.23209	0.397174
UBA	0.15383451	0.15378
Zenith bank	0.133124999	0.137543

Investing in the stock market always bears some risk which may be large or small, depending on the variance of the returns of the stock. A stock that has large variance may make you rich but may also make you poor; therefore risk is associated with variance. A risk averse investor will like to invest in a stock with (most) minimum risk.

From table 3.2 above we observed that Var(FB)<Var(ZB)<Var(DB)<Var(UB)<Var(EB)

If the investor is risk averse, he would invest more of his fund in Firstbank (FB), followed by Zenith bank, then Diamond bank and in that order. If the investor is one that likes to take risk, (investor that needs more returns) he would invest more of his fund in Ecobank followed by UBA etc. This is because Ecobank has the highest variance.

Another question that follows is what proportion of fund he would invest in each of these stocks in order to have the maximum returns associated with minimum risk. This question will be answered in section 4.6

4.6 Portfolio Management with two Stocks

The proportions of funds to be invested into any of the two stocks were calculated using equation (2.4) and (2.5) and the results are as shown below

Table 3.3a: portfolio with two stocks

	W_A (FB)	W_B (DB)	W_C (EB)	W_D (UB)	W_E (ZB)
FB,DB	0.954644084	0.045355916			
FB,EB	0.812201982		0.187798018		
FB,UB	0.940020773			0.059979227	
FB,ZB	O.791609364				0.208390636
DB,EB		0.688417353	0.311582647		
DB,UB		0.522602844		0.477397156	
DB,ZB		0.29679143			0.70320857
EB,UB			0.310904492	0.689095508	
EB,ZB			0.246666567		0.753333433
UB,ZB				0.235082	0.764918

Table 3.3b: Portfolio with Three Stocks

	w_A(FB)	w_B (DB)	w_C (EB)	w_D (UB)	w_E (ZB)
FB, DB, EB	0.779744579	0.032895599	0.187359822		
FB, DB, UB	0.935075	0.011071		0.053854	
FB, DB,ZB	0.259831	0.320543			0.419626
DB, EB, UB		0.4400776514	0.284400019	0.314823466	
DB, EB, ZB		0.213057	0.253586		0.533357
EB, UB, ZB			0.258113211	0.137478601	0.604408188

Table 3.3c: Portfolio with four stocks

	FB(W_A)	DB(W_B)	EB(W_C)	UB(W_D)	ZB(W_E)
FB,DB,EB,UB	0.083097782	0.042217127	0.3399467667	0.475217423	
FB,DB,EB,ZB	0.67313	0.07198	0.181364		0.070343
DB,EB,UB,ZB		0.065226	0.184672	0.129955	0.620147

If an investor wishes to invest in First bank and Diamond bank, he will invest 95.5% of his capital into First bank and 4.5% of the capital in Diamond bank, the variance of the portfolio will be minimized and equal to 0.010412805 for log return. If he chooses to invest in First bank and Eco bank, the proportion of fund to be invested into First bank is 81.2% while 18.8% goes to Eco bank, and in that order.

If an investor wishes to invest in three stocks out of the N listed companies, the proportion of capital to invest in each of these stocks in order to minimize risk are as shown below. Suppose he decides to invest in First bank, Diamond bank and Eco bank, the best proportion of capital to invest in each of the three are First bank 78%, Diamond bank 3.3% and Eco bank 18.7%. For Diamond bank, Eco bank and Zenith bank, 21.3% of the capital goes to Diamond bank, 25.4% for Eco bank and 53.3% should be invest into Zenith bank, see table 3.3b for details.

An investor may also be interested in investing in four out of the N available companies. The proportion of capital to be invested in each of the four companies for a minimum risk depends on the investor's choice of companies. Suppose he decides to invest in Diamond bank, Eco bank, UBA and Zenith bank, the proportion of capital to be invested in each of the four companies with minimum risk are 6.5% of his capital for Diamond bank, 18.5% for Eco bank, 13% for UBA and 62% for Zenith bank.

4.7 Covariance of the Return

The covariances of the returns were calculated using equation (1.7) and (1.8) and the results are displayed below.

In the case of portfolio with two stocks, we observed that the covariance of First bank and Eco bank, Diamond bank and Eco bank, UBA and Eco bank, Zenith bank and Eco bank are negative, which implies that the returns of the stocks move in opposite direction. While the return of one is appreciating the other is depreciating. Those that have positive covariance imply that the stocks move in the same direction. One may conclude that the negative covariance was as a result of the high volatility of Eco bank. The graph of closing prices of Eco bank also shows that.

In the case of portfolio with three stocks their covariance are all positive.

Table 3.4a Covariance of Return for Two Stocks

	Log return	Return at time (t)
FB, DB	0.009809658	0.010111234
FB, EB	- 0.00261905	-0.00706033
FB, UB	0.009540207	0.00950717
FB, ZB	0.007889947	0.007881237
DB, EB	- 0.002324449	- 0.00556552
DB, UB	0.0172221	0.019664531
DB, ZB	0.0137895	0.015928186
EB, UB	- 0.0011626	- 0.005553311
EB, ZB	- 0.0027672	- 0.006264598
UB, ZB	0.0150855	0.016540385

Table 3.4b Covariance of Return for Three Stocks

	Log return	Return at time (t)
FB, DB, EB	0.000602877	0.00107621
FB, DB, UB	0.000488489	0.001129247
FB, DB, ZB	0.000203541	0.000657
DB, EB, UB	0.000828	0.001339542
DB, EB, ZB	0.001028	0.001402274
EB, UB, ZB	0.001038	0.001308

Table 3.4c Covariance of Return for Four Stocks

	Log return	Return at time (t)
FB, DB, EB, UB	- 6.38328E-05	- 0.00021599
FB, DB, EB, ZB	- 9.34733E-05	- 0.000211173
DB, EB, UB, ZB	1.321E-05	- 0.000146281

4.8 Expected Return of the Portfolio

The expected return of the portfolio is calculated using equation (3.1) and the data is displayed below

Table 3.5a: Expected Return of the Portfolio for Two Stocks

	Log expected Return	Expected return of time (t)
FB, DB	- 0.0220358	- 0.01473222
FB, EB	- 0.0076031	0.0224022
FB, UB	- 0.0213472	- 0.015655249
FB, ZB	- 0.0194795	- 0.013461548
DB, EB	-.0061143	0.014048153
DB, UB	- 0.014434	- 0.002194027
DB, ZB	- 0.0107358	- 0.000645721
EB, UB	- 0.0202412	- 0.000568034
ED, ZD	- 0.005082	0.003592854
UB, ZD	- 0.0160907	- 0.0063793227

Table 3.5b Expected return of Portfolio for Three Stocks

	Log expected return	Return at time (t)
FB, DB, EB	- 0.0183628	- 0.012324485
FB, DB, UB	- 0.0211402	- 0.01540809
FB, DB, ZB	0.0175568	0.066444869
DB, EB, UB	- 0.0124417	0.006842228
DB, EB, ZB	- 0.0104606	0.008682388
EB, UB, ZB	- 0.0139164	0.002719131

Table 3.5c: Expected Return of the Portfolio for Four Stocks

	Log expected return	Expected return at time (t)
FB, DB, EB, UB	- 0.01764104	0.00373351
FB, DB, EB, ZB	-0.01724139	-0.005417968
DB, EB, UB, ZB	- 0.0135521	0.005614959

From table 3.5a (that is portfolio with the two stocks) combining First bank and Eco bank will give the highest expected return, followed by Eco bank and Zenith bank, while First bank and Diamond bank will give the lowest expected return. This is because stocks with uncertain returns give high expected return than stocks with certain return. Eco bank has the most uncertain return.
In table 3.5b, we observed that the combination of Diamond bank, Eco bank and Zenith bank give the highest return while First Bank, Diamond bank and Eco bank have lowest return.
The table 3.5c the combination of Diamond bank, Eco bank, UBA and Zenith bank give the highest return.

4.9 Variance of the Portfolio

The variance of the portfolio is calculated using equation (3.20) and the result is as shown below;

Table 3.6a Variance of the portfolio for two stocks

Variance	Variance (Log)	Variance at time (t)
FB, DB	0.010412805	0.009924906
FB, EB	0.035533868	0.104061324
FB, UB	0.010387404	0.009899404
FB, ZB	0.00989933	0.009605226
DB, EB	0.015183507	0.033892626
DB, UB	0.020297944	0.022330719
DB, EB	0.01655041	0.017803183
EB, UB	0.015946027	0.025979313
EB, ZB	0.012306616	0.079305781
UB,ZB	0.017102415	0.017801094

Table 3.6b: Variance of the portfolio for three stocks

Variance	Variance (Log)	Variance of time (t)
FB, DB, EB	0.007973662	0.011292608
FB, DB, ZB	0.010385599	0.009903645
FB, DB, ZB	0.013253072	0.028656083
DB, EB, UB	0.014125976	0.023620227
DB, EB, ZB	0.011688515	0.019047325
EB, UB, ZB	0.01207117	0.019428468

Table 3.6c: Variance of the portfolio for four stocks

Variance	Variance (Log)	Variance of time (t)
FB, DB, EB, UB	0.014872885	0.03145531
FB, DB, EB, ZB	0.007822035	0.01103098
DB, EB, UB, ZB	0.012244096	0.016278135

From table 3.6a above, it can be seen that combining First bank and Eco bank has the highest variance (risk), followed by Eco bank and UBA. The combination of First bank and Zenith bank has the lowest variance. This is because stocks with high volatility are associated with higher variance while those with low volatility are associated with lower variance.

In table 3.6b, the combination of Diamond bank, Eco bank, and UBA gives the highest variance followed by First bank, Diamond bank and Zenith bank while the combination of First bank, Diamond bank and UBA has the lowest variance, while in table 3.6c, the combination of Diamond bank, Eco bank, UBA and Zenith has the lowest risk (variance). This indicates the low volatility. We can also observe from the above tables that diversification reduces the risk of investor. For example Var ($W_AR_A + W_BR_B + W_CR_C + W_DR_D$) <Var (Fb) <Var ($Z_B$) <Var($D_B$) <Var($U_B$) <Var ($E_b$)
This implies that individual securities have higher variance than when they are combined to form a single portfolio. This is in conformity with Bill's (2004) assertion that as the number of the stocks contained in a portfolio increases, the variance of the portfolio decreases (ie as n $\rightarrow\alpha$, Vp$\rightarrow$ 0).

4.10 Portfolio Risk

Level of risk reduction for portfolio with two stocks.

	Log reduction			**Reduction at time (t)**		
	$\sigma^2 + \sigma^2$	Vp	Level of reduction	$\sigma_i^2 + \sigma_j^2$	Vp	Level of reduction
FB, DB	0.033549194	0.010412805	2.3%	0.0404538	0.009924506	3.05%
FB, EB	O.O64307461	0.035533868	2.88%	0.167636	0.104061324	6.36%
FB, UB	0.034106518	0.010387404	2.37%	0.033537	0.00989404	2.36%
FB, ZB	0.028163726	0.00989933	1.83%	0.028807	0.009605226	1.92%
DB, EB	0.076973733	0.0515183507	2.54%	0.1883118	0.033892626	1.54%
DB, UB	0.04677279	0.020297944	2.65%	0.0542128	0.02232079	3.19%
DB, ZB	0.0040829998	0.01655041	2.43%	0.0494828	0.017803183	3.17%
EB, UB	0.077531057	0.015946027	6.16%	0.181395	0.025979313	1.55%
EB, ZB	0.071588265	0.012306616	5.92%	0.176665	0.079305781	1.57%
UB, ZB	O.041387322	0.17102415	1.30%	0.042566	0.017801094	2.48%

Table 3.7b Level of risk reduction for portfolio with three stocks

	$\sigma_i^2 + \sigma_j^2 + \sigma_k^2$	Vp	Log Reduction	$\sigma_i^2 + \sigma_j^2 + \sigma_k^2$	Vp	Time(t)
EB, DB, EB	0.08741515	0.007974	7.94%	0.198201123	0.011292608	18.69%
FB, DB, UB	0.05721425	0.010386	4.68%	0.06410222	0.009903645	5.42%
FB, DB, ZB	0.05127146	0.013253	3.80%	0.05937205	0.028656083	3.07%
DB, EB, UB	0.10063875	0.014126	8.65%	0.2119600	0.023620227	18.83%
DB, EB, ZB	0.09469596	0.011689	8.39%	0.207229836	0.019047325	18.82%
EB, UB, ZB	0.09525328	0.012071	8.32%	0.200313185	0.019428468	18.09%

Table 3.7c Level of risk reduction for portfolio with four stocks

	$\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2$	**Vp**	**Reduction**	$\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2$	**Vp**	**Reduction**
FB, DB, EB, UB	0.11108021	0.0148732885	9.62%	0.221849273	0.03145531	19.04%
FB, DB, EB,ZB	0.10513742	0.007822035	9.73%	0.217119104	0.1103089	10.68%
DB, EB, UB, ZB	0.11836101	0.012244096	10.61%	0.230877987	0.016278135	21.46%

From table 3.7 above, it can be observed that the variance of the portfolio is less than the combination of variance of the individual securities contained in the portfolio. This is because diversification has reduced the risk of these individual securities. We observed that as the number of individual securities combined in a given portfolio increases, the lower the risk of the investor.

4.11 Correlation Coefficient

Correlation coefficient is calculated using equation (1.9) and the result displayed below

Table 3.8a: Correlation coefficient between two stocks

	Log Correlation	**Correlation at time (t)**
FB, DB	0.6315309	0.581582
FB, EB	- 0.110435	- 0.178757
FB, UB	0.6069087	0.621685
FB, ZB	0.5763322	0.576202
DB, EB	- 0.065885	- 0.080152
DB, UB	0.7364672	0.731432
DB, ZB	0.6814106	0.662397
EB, UB	- 0.032563	- 0.090923
EB, ZB	- 0.089562	- 0.114677
UB, ZB	0.7366259	0.782005

Table 3.8b: Correlation coefficient between three stocks

	Log Correlation	**Correlation at time (t)**
FB, DB, EB	0.009575064	0.0345672
FB, DB, UB	0.0051422381	0.01404346
FB, DB, ZB	0.001854245	0.007307
DB, EB, UB	0.00894599	0.019254026
DB, EB, ZB	0.009615	0.020155698
EB,UB,ZB	0.004212	0.003203

Table 3.8c: Correlation coefficient between four stocks

	Log correlation	Correlation at time (t)
FB, DB, EB, UB	- 0.000155959	- 0.00021599
FB, DB, EB, ZB	- 0.000197633	- 0.000211173
DB, EB, UB, ZB	1.9006 E-05	- 0.000146281

Correlation coefficient measures the strength (degree) of association between the variables. From the table above, the portfolios with positive correction implies that as the price of one stock increase the other increases as well. That is they move in the same direction. Those with negative correlation imply that as the price of one increase the other decreases, that is they move in opposite direction.

Summary, Conclusion and Recommendation

5.1 Summary

We have presented a brief theory on portfolio risk management and how it works with three examples (with two, three and four stocks) using real market data. For this study all the analyses were done using Microsoft excel. However, it is also important to note here that the same analysis can be done using any other statistical software like matlabs and R.

Among the five stocks Eco bank has the highest unstable price (it has the greatest fluctuations) and First bank has the most stable price. From the mean and variance of the return were calculated , we observed that First bank has the lowest mean return of -0.0211 with a variance of 0.01044 while Eco bank has the highest mean return of -0.00936 with a variance of 0.0539 (from log return). A risk adverse investor will like to invest where there is minimum risk. Therefore his choice of investment will go in this order FB, ZB, DB, UB, EB. An investor that is risk tolerance may wish to go for Eco bank.

The proportion of fund (weight) to be invested in each of the securities based on their contributions was calculated. It was observed that when stock with higher variance is combined with stock with low variance the covariance of the portfolio will be reduced compared with the individual variance of each security. But when two stocks with high variance are combined their covariance will also be high and this will not provide much diversification. Expected return of portfolio was also calculated. In the case of portfolio with two stocks, when First bank combines with zenith bank gives the lowest variance of 1.0% with a return of -1.95% while First bank combining with Eco bank has the highest variance of 3.6% with a return of 0.76%.

In the case of portfolio with three stocks, we observed that First bank, Diamond bank and Eco bank have a variance of 0.797% with expected return of -1.836% while Diamond bank, Eco bank and UBA combined have variance of 1.41% with expected return of 1,244%. Also combining First bank, Diamond bank, Eco bank and Zenith bank the variance is 0.782% with expected return of -1.704% and when First bank, Diamond bank, Eco bank and UBA combined the variance is1.49% with expected return of -1.764%.

5.2 Conclusion

From the above result as the portfolio of an investor increases the lower the variance and the higher the return. This implies that diversification increases return and minimizes the risk of investors. The results obtained from portfolio possibility curve shows that for a risk averse investor the proportion of fund to be allocated to various securities is best obtained at the point where the variance is minimum.

5.3 Recommendation

I recommend that other researchers should use other approach like Lagrange multipliers, or polynomial goal programming (PGP) to work on this data in order to find out if the result will be the same.

Reference

Alexander C. and A. Dimilriu, (2005) Indexing and statistical arbitrage. *The Journal of Portfolio Management, 31*, 50-64

Benninga S. (2007) Shrinking is Better, *Journal of Portfolio Management*, 33 (4), 56-63.

Christou N. (2008) Enhancing the teaching of statistics: portfolio theory, an application of statistics in finance. *Journal of Statistics Education*, 16 (3),1-11

Chunhachinda P. (1997) Portfolio selection and skewness: Evidence from international stock markets. *Journal of Banking and Finance*, 21,143-167. (doi:10.1016/s0378-4266(96)00032-5)

Duncan B. (2008) Portfolio Risk minimization using historical data. Electronic copy available at**: http/www.wpi.edu/pubs/E-project.**

Harvey et al (2004) *Portfolio selection with higher moments*. Duke University, working paper.(1-51)

Jurezenk E. (2006) Theoretical foundations of assets allocation and pricing models with higher order moments in *Multi-moment Asset Allocation and Pricing Models*. Ed. John Wiley, 51-66.

Klaus G. (2010) The stock selection. *Journal of Applied Finance and Banking*, 1 (1), 142-162.

Lai T.Y. (1991) Portfolio selection with skewness: Multiple-objective approach, *Review of Quantitative Finance and Accounting*, 1,293-305.(doi:10.1007/BF02408382)

Markowitz H.M. (1952) Portfolio selection. *Journal of Applied Finance and Banking*,7,77-91

Prakash, A. (2003). Selecting a portfolio with skewness: recent Evidence from US. European, and Latin America Equity markets, *Journal of Banking and Finance*, 27,1375-1390. (doi:10.1016/S0378-4266(02)00261-3)

Roll (1979) Testing a portfolio of Ex Ante, mean-variance Efficiency. *TIMS Studies in the Management Science*, 11, 135-149.

Rustern B. (2009) Portfolio decision with higher order moments. COMISEF: Working papers. series 22.

Tayi G. Leonard P. (1988) Banking balance sheet management: An alternative multi-objective model, *Journal of the Operational Research Society*,39, 401-410.

Wang S. Xia Y. (2002). *Portfolio selection and Assets Pricing*. Springer-Verlag, Berlin.

Appendix I

MONTHLY STOCK PRICES.						
NO	MONTH	FIRST BANK	DIAMOND BANK	ECOBANK	UBA	ZENITH BANK
1	JANUARY	35.08	9.28	5.48	28.73	27.68
2	FEBUNARY	38.89	11.55	7.08	37.73	32.79
3	MARCH	38.07	10.70	6.17	37.99	36.14
4	APRIL	40.11	10.70	6.64	37.99	45.42
5	MAY	40.40	10.86	7.71	38.55	49.63
6	JUNE	40.40	19.23	8.82	46.45	60.03
7	JULY	41.87	19.52	8.75	53.91	63.50
8	AUGUST	47.14	18.65	8.92	53.29	56.41
9	SEPTEMBER	40.81	18.00	9.08	53.16	45.96
10	October	39.41	17.97	8.27	52.92	44.43
11	NOVEMBER	41.50	18.54	7.94	53.19	46.09
12	DECEMBER	42.70	18.89	7.95	49.02	46.09
13	JANUARY	42.38	22.25	7.95	50.05	46.09
14	FEBUNAY	49.03	21.38	7.95	50.05	49.43
15	MARCH	47.66	20.29	7.95	49.30	49.60
16	APRIL	43.60	19.41	7.95	52.39	48.17
17	MAY	44.66	18.47	9.38	58.41	48.71
18	JUNE	39.34	16.08	7.90	33.60	43.93
19	JULY	42.37	15.61	7.47	32.04	41.56
20	AUGUST	31.35	13.08	5.83	28.03	39.75
21	SEPTEMBER	30.55	12.00	7.81	27.97	40.32
22	OCTOBER	24.79	10.16	28.00	22.41	33.80
23	NOVEMBER	24.00	8.45	28.00	16.92	25.15
24	DECEMBER	20.24	7.11	28.00	13.92	20.56
25	JANUARY	17.94	5.42	28.00	10.45	16.53
26	FEBUNAY	17.04	5.08	28.00	8.89	15.05
27	MARCH	15.74	4.63	28.00	8.24	13.02
28	APRIL	14.92	4.71	28.00	8.85	13.28
29	MAY	18.12	6.99	28.00	14.13	20.16
30	JUNE	22.35	9.28	28.00	14.75	17.68
31	JULY	18.27	7.77	28.00	12.42	13.24
32	AUGUST	14.83	6.71	28.00	11.82	12.44
33	SEPTEMBER	14.58	7.15	24.50	12.11	12.46
34	OCTOBER	15.02	7.86	20.30	13.13	14.64
35	NOVEMBER	14.15	7.32	16.10	11.76	13.54
36	DECEMBER	14.13	7.30	12.10	11.11	13.31
37	JANUARY	14.63	7.65	9.67	11.29	15.08
38	FEBUNAY	14.90	9.13	5.90	13.31	15.78
39	MARCH	15.43	9.22	6.42	14.08	17.18
40	APRIL	16.57	9.01	6.28	14.95	16.32
41	MAY	14.78	8.29	5.55	12.46	14.99
42	JUNE	13.76	7.83	4.98	11.05	13.48
43	JULY	13.24	7.40	4.39	10.30	13.13

44	AUGUST	12.86	7.02	4.62	9.96	13.40
45	SEPTEMBER	11.69	5.98	4.01	9.15	12.10
46	OCTOBER	12.36	7.35	3.80	9.59	13.33
47	NOVEMBER	12.93	7.71	3.69	9.12	14.57
48	DECEMBER	13.01	7.46	3.53	8.81	14.81

APPENDIX II

Fig 1

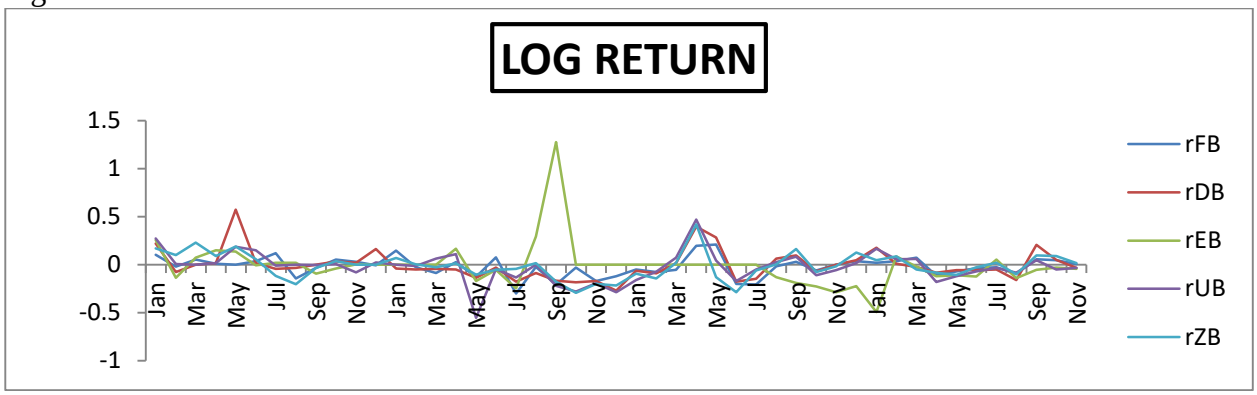


Fig 2

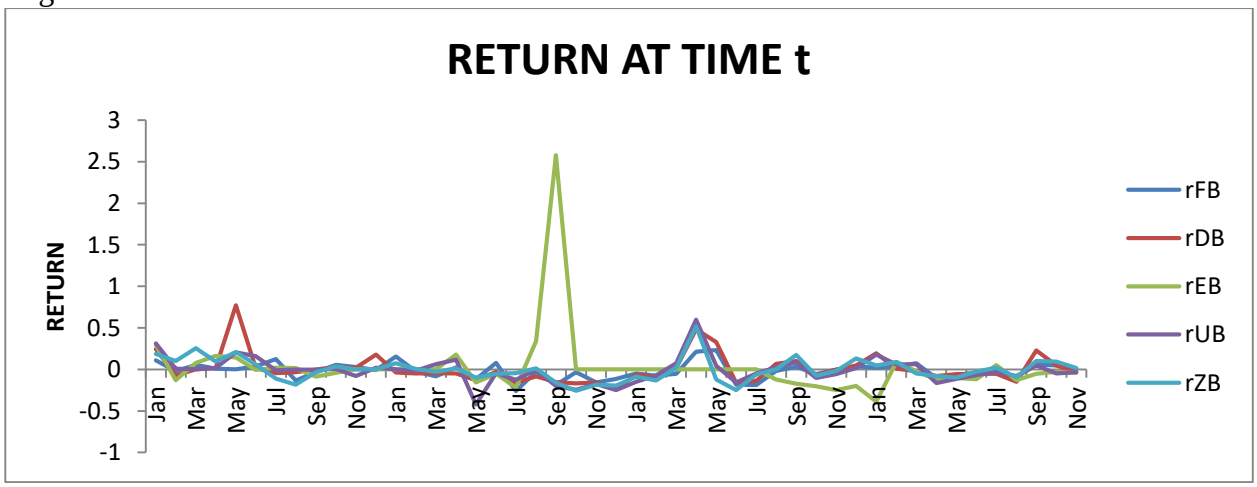
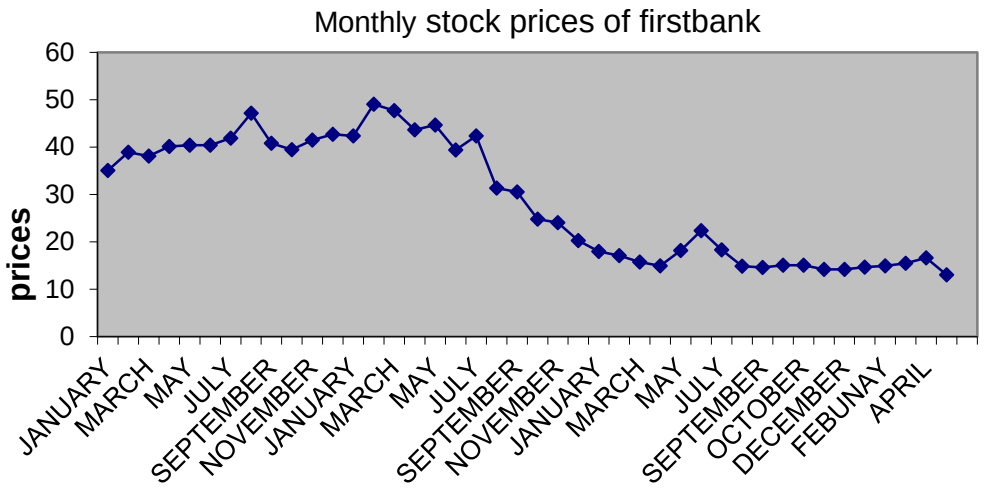


Fig 1a:



Monthly prices of FB

Fig 1b: Monthly prices of Diamond bank

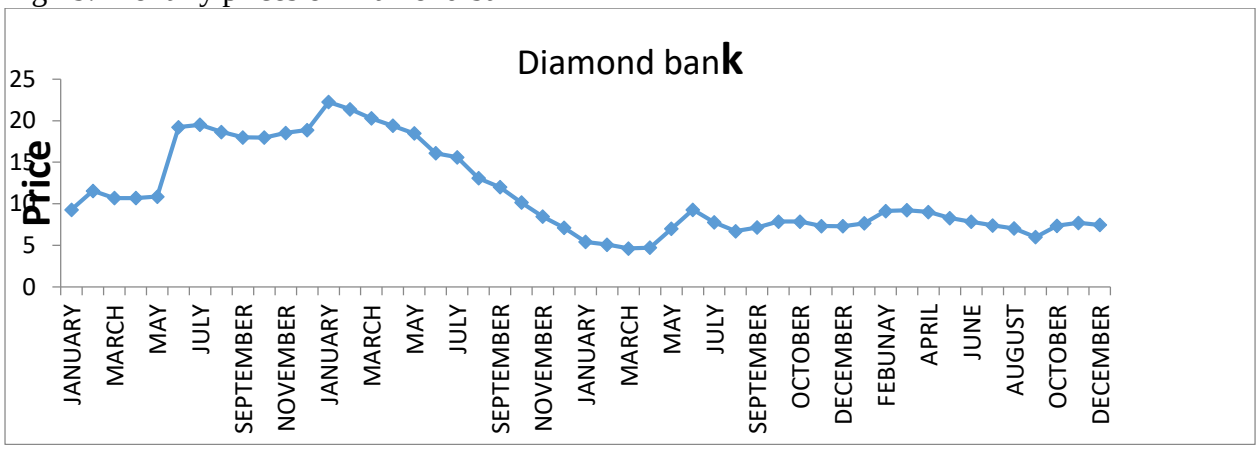


Fig 1c: Monthly prices of Ecobank

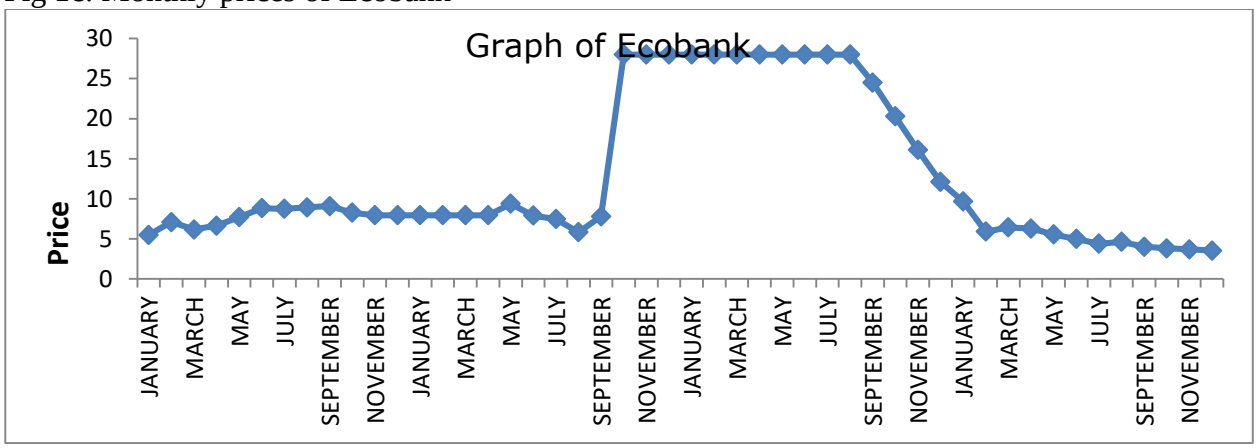


Fig 1d: Monthly prices of UBA

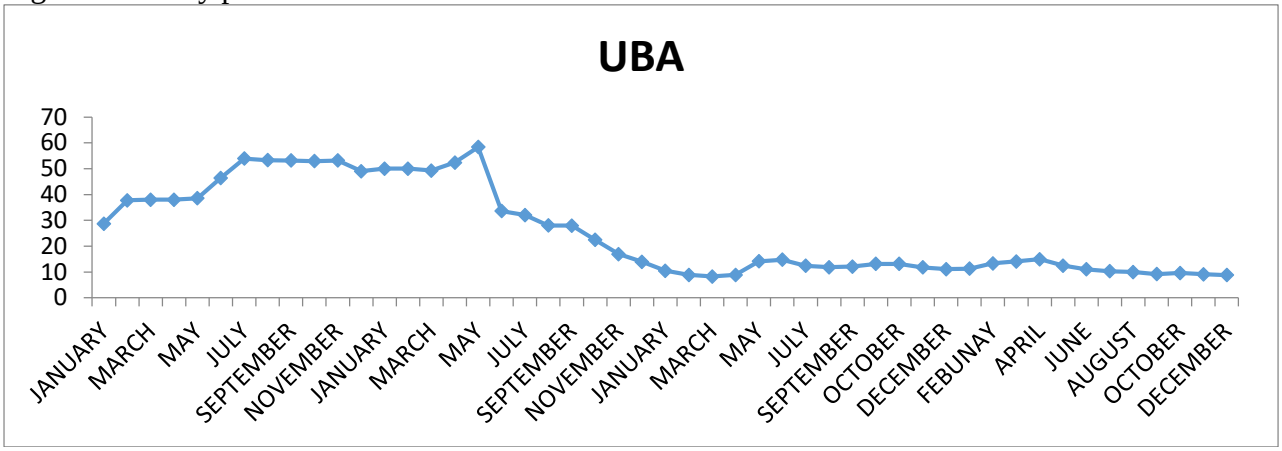


Fig 1e: Monthly prices of Zenith bank